University of Diyala
College of Engineering
Dep. of Computer Engineering

Fuzzy Logic [Principles & Applications]

Asst.Lect Hussein.s. Radhi

OUTLINE

- Fundamental fuzzy concepts
- Fuzzy propositional and predicate logic
- Fuzzification
- Defuzzification
- Fuzzy control systems
- Types of fuzzy algorithms
- Applications of fuzzy logic

INTRODUCTION

- Fuzzy concepts first introduced by Zadeh in the 1960s and 70s
- Traditional computational logic and set theory is all about
- true or false
- zero or one
- in or out (in terms of set membership)
- black or white (no grey)
- Not the case with fuzzy logic and fuzzy sets!

BASIC CONCEPTS

Approximation ("granulation")

A color can be described precisely using RGB values, or it can be approximately described as "red", "blue", etc.

DEGREE ("GRADUATION")

Two different colors may both be described as "red", but one is considered to be more red than the other

Fuzzy logic attempts to reflect the human way of thinking

TERMINOLOGY

Fuzzy set

- A set X in which each element y has a grade of membership μX(y) in the range 0 to 1, i.e. set membership may be partial e.g. if *cold* is a fuzzy set, exact temperature values might be mapped to the fuzzy set as follows:
- 15 degrees → 0.2 (slightly cold)
- 10 degrees → 0.5 (quite cold)
- 0 degrees → 1 (totally cold)

Fuzzy Relation

- Relationships can also be expressed on a scale of 0 to 1
- e.g. degree of *resemblance* between two people
-

Fuzzy variable

- Variable with (labels of) fuzzy sets as its values

Linguistic variable

Fuzzy variable with values that are words or sentences in a language e.g. variable *colour* with values *red*, *blue*, *yellow*, *green*...

Linguistic hedge

Term used as a modifier for basic terms in linguistic values e.g. words such as *very*, *a bit*, *rather*, *somewhat*, etc.

Formal Fuzzy Logic

- Fuzzy logic can be seen as an extension of ordinary logic, where the main difference is that we use fuzzy sets for the membership of a variable
- We can have fuzzy propositional logic and fuzzy predicate logic
- Fuzzy logic can have many advantages over ordinary logic in areas like artificial intelligence where a simple true/false statement is insufficient

TRADITIONAL LOGIC

Propositional logic:

- Propositional logic is a formal system that uses true statements to form or prove other true statements
- There are two types of sentences: simple sentences and compound sentences
- Simple sentences are propositional constants; statements that are either true or false
- Compound sentences are formed from simpler sentences by using negations $\neg$, conjunctions $\wedge$, disjunctions $\vee$, implications $\Rightarrow$, reductions $\Leftarrow$, and equivalences $\Leftrightarrow$
- *Predicate logic:*
- Onto propositional logic, this adds the ability to quantify variables, so we can manipulate statements about all or some things

FORMAL FUZZY LOGIC

Fuzzy Propositional Logic

- Like ordinary propositional logic, we introduce propositional variables, truth-functional connectives, and a propositional constant 0
- Some of these include:
- Monoidal t-norm-based propositional fuzzy logic
- Basic propositional fuzzy logic
- Łukasiewicz fuzzy logic
- Gödel fuzzy logic
- Product fuzzy logic
- Rational Pavelka logic

FUZZY PREDICATE LOGIC

- These extend fuzzy propositional logic by adding universal and existential quantifiers in a manner similar to the way that predicate logic is created from propositional logic

Simple Fuzzy Operators

- As described by Zadeh (1973)…
- NOT X = 1 – μX(y)

  e.g. 0.8 cold → (1 – 0.8) = 0.2 NOT cold
- X OR Y (union) = max(μX(y), μY(y))

  e.g. 0.8 cold, 0.5 rainy → 0.8 cold OR rainy
- X AND Y (intersection) = min(μX(y), μY(y))

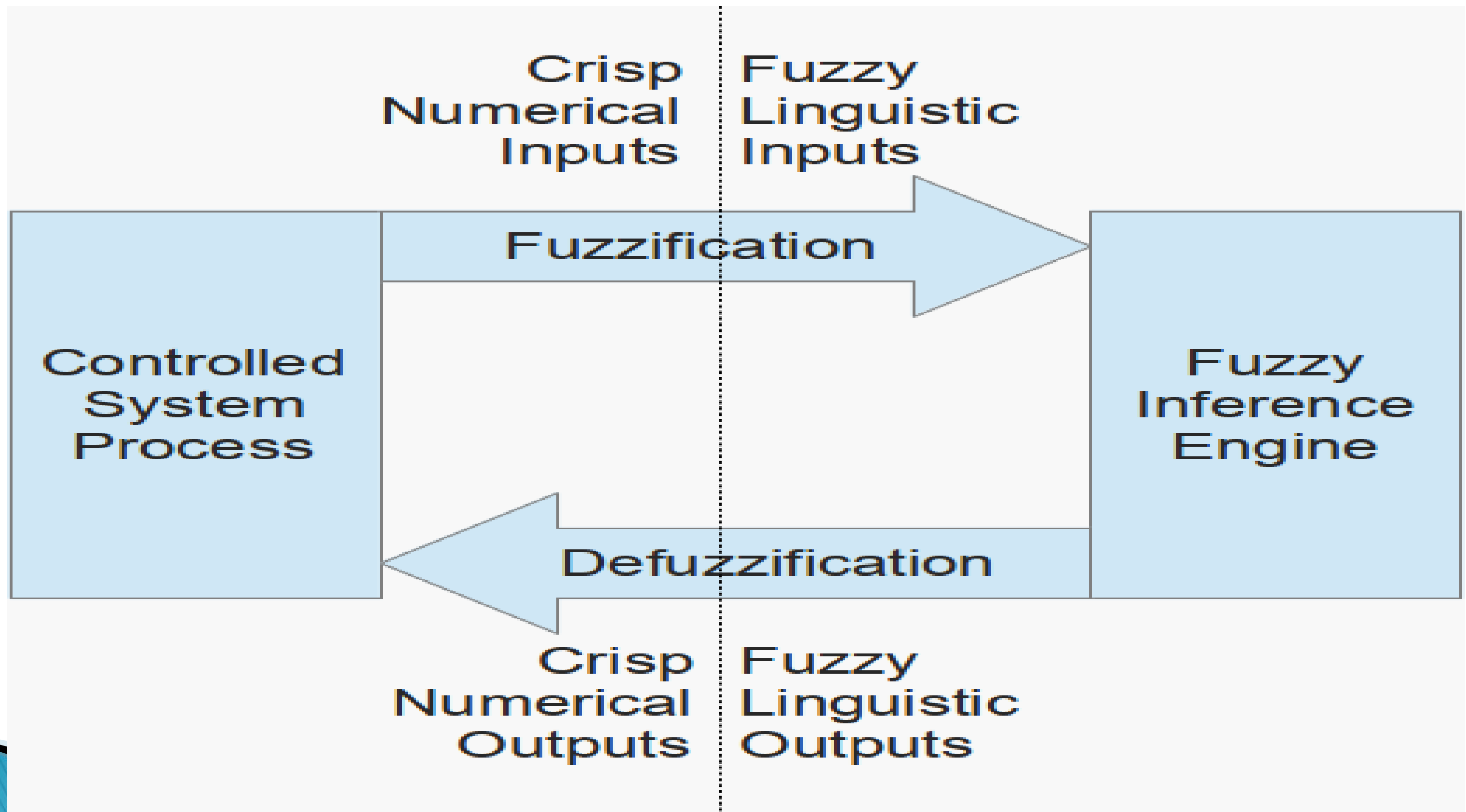
  e.g. 0.9 hot, 0.7 humid → 0.7 hot AND humid

Alternative Interpretations of AND and OR

- Zadeh's definition of AND used the Gödel t-norm, but other definitions are possible using different t-norms
- Common examples:
- Product t-norm: $\mu X(y) * \mu Y(y)$ e.g. 0.9 hot, 0.7 humid → 0.63 hot AND humid
- Lukasiewicz t-norm: $\max(\mu X(y) + \mu Y(y) - 1, 0)$ e.g. 0.9 hot, 0.7 humid → 0.6 hot AND humid
- Similar possibilities for OR using corresponding t-conorms:
- Product t-conorm: $\mu X(y) + \mu Y(y) - \mu X(y) * \mu Y(y)$ e.g. 0.8 cold, 0.5 rainy → 0.9 cold OR rainy
- Lukasiewicz t-conorm: $\min(\mu X(y) + \mu Y(y), 1)$ e.g. 0.8 cold, 0.5 rainy → 1 cold OR rainy

FUZZY SYSTEM OVERVIEW

BLOCK DAIGRAM



*When making inferences, we want to clump the continuous numerical values into sets

*Unlike Boolean logic, fuzzy logic uses fuzzy sets rather than crisp sets to determine the membership of a variable

*This allows values to have a degree of membership with a set, which denotes the extent to which a proposition is true

*The membership function may be triangular, trapezoidal, Gaussian or any other shape

FUZZIFICATION

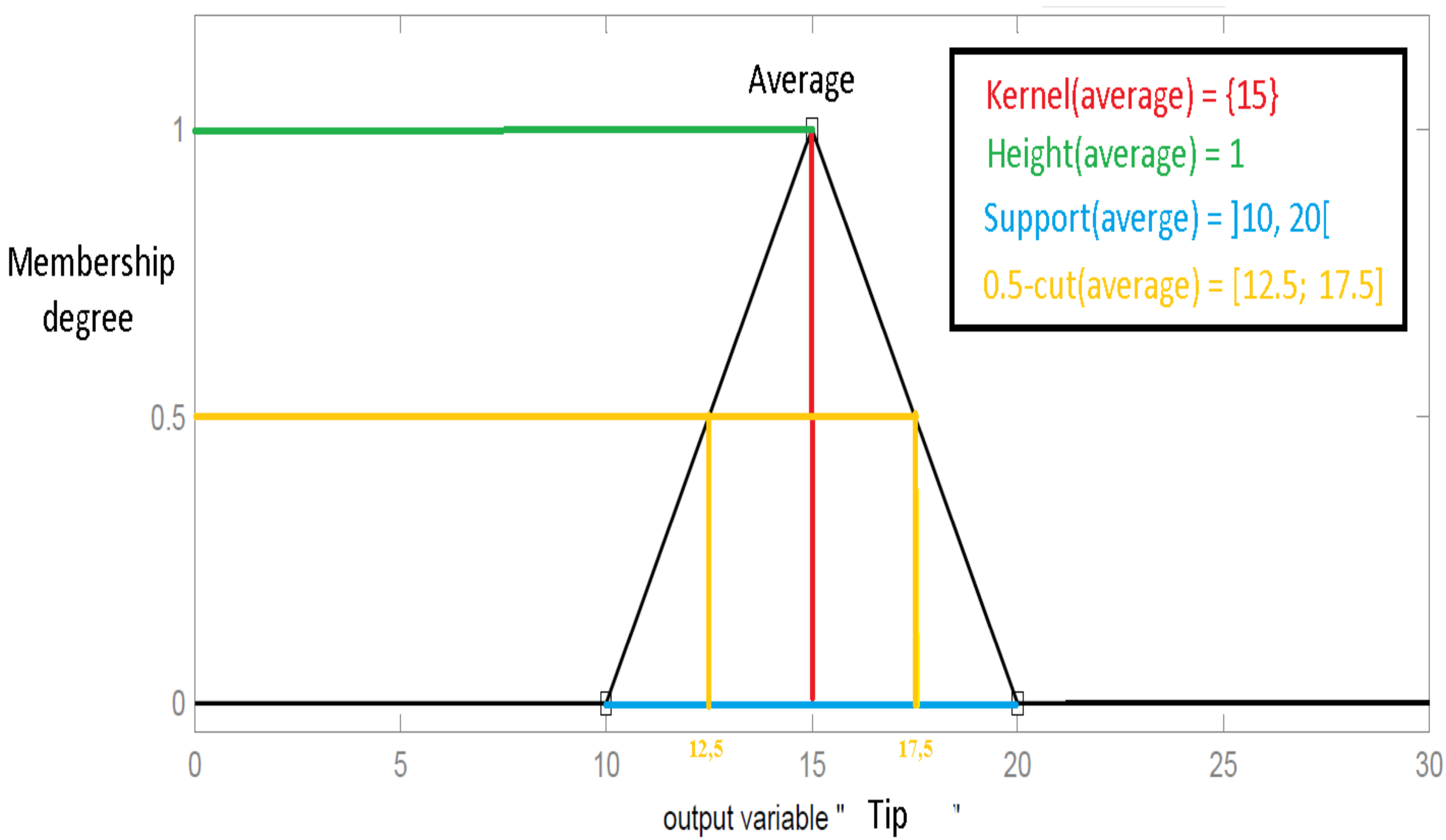
- To apply fuzzy inference, we need our input to be in linguistic values
- These linguistic values are represented by the degree of membership in the fuzzy sets
- The process of translating the measured numerical values into fuzzy linguistic values is called fuzzification
- In other words, fuzzification is where membership functions are applied, and the degree of membership is determined

MEMBERSHIP FUNCTIONS

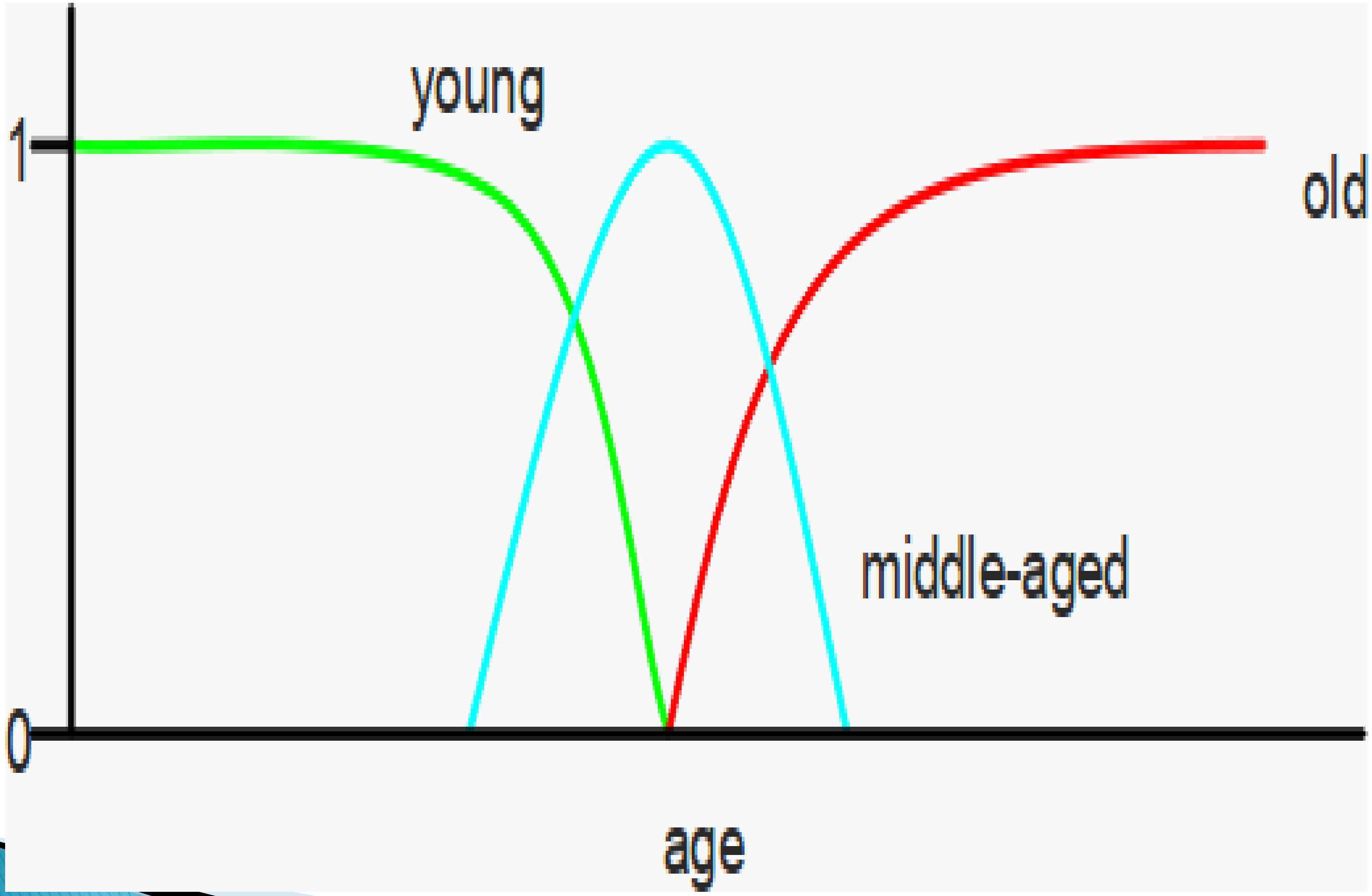
There are largely four types of fuzzifiers:

- singleton fuzzifier,
- Gaussian fuzzifier,
- Trapezoidal or triangular fuzzifier
- Triangular fuzzifier

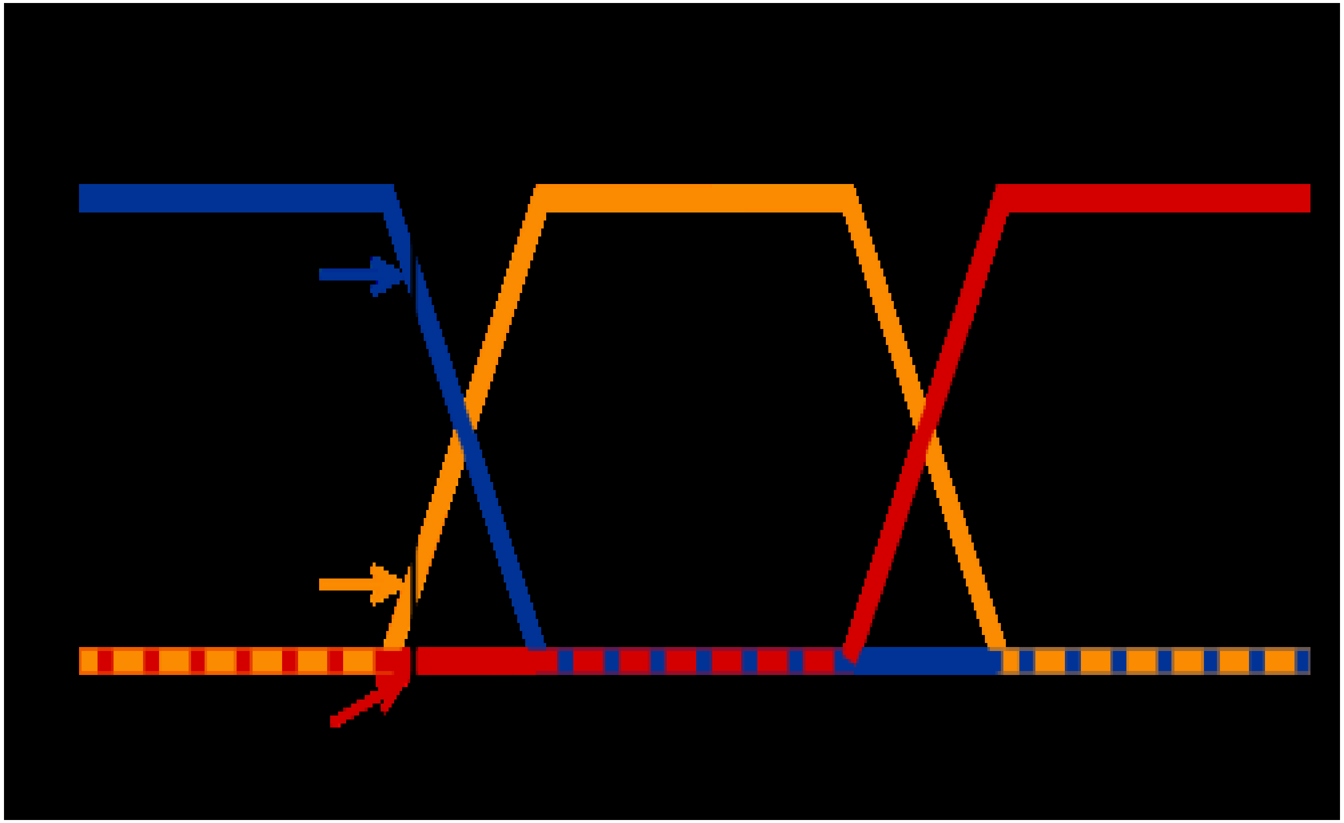
A membership function with properties displayed



Gaussian



Trapezoidal



DEFUZZIFICATION

.Defuzzification is the process of producing a quantifiable result in fuzzy logic

.The fuzzy inference will output a fuzzy result, described in terms of degrees of membership of the fuzzy sets

.Defuzzification interprets the membership degrees in the fuzzy sets into a specific action or real-value

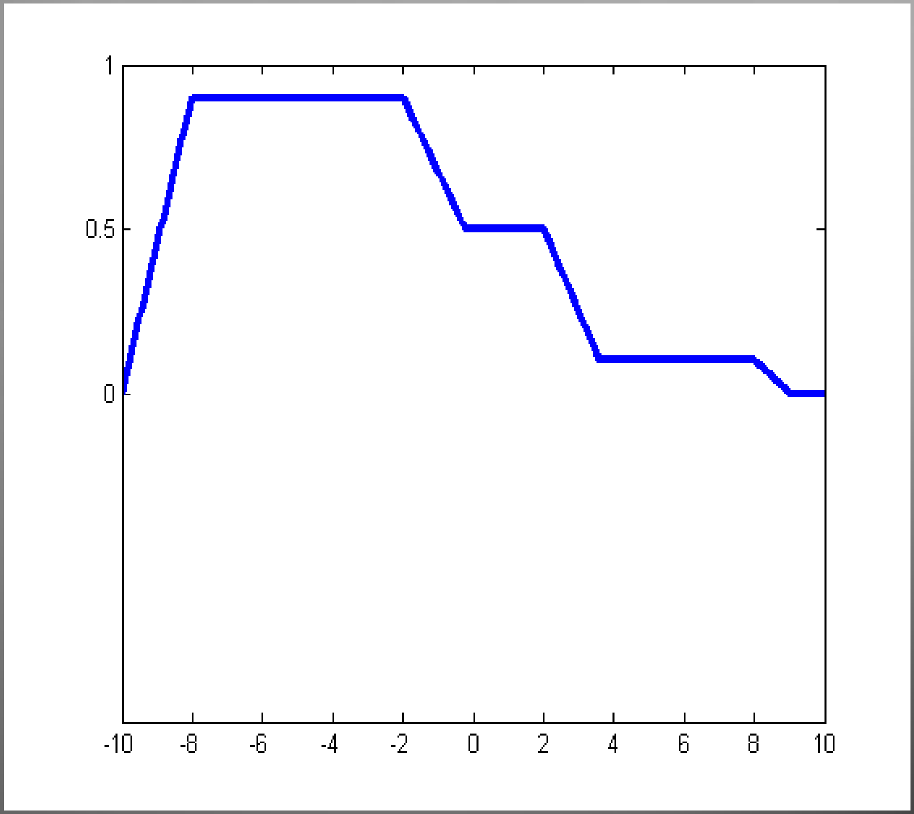
METHODS OF DEFUZZIFICATION

*There are many methods for defuzzification
* One of the more common types of defuzzification technique is the maximum defuzzification techniques. These select the output with the highest membership function
They include:
1- First of maximum
2- Middle of maximum
3- Last of maximum
4-Mean of maxima
5- Random choice of maximum

Given the fuzzy output:

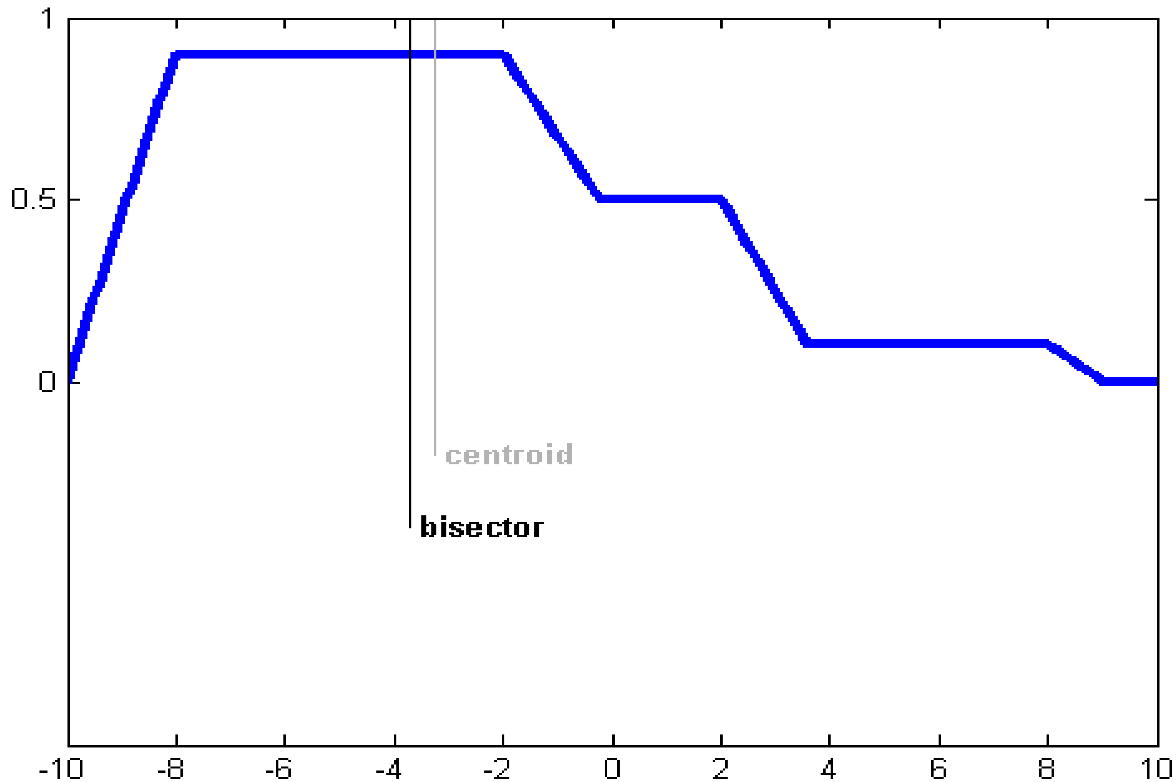
1-The first of maximum, middle of maximum, and last of maximum would be -2, -5, and -8 respectively as seen in the diagram

2-The mean would give the same result as middle unless there is more than one plateau with the maximum value



Common Method is Centre of Gravity

Calculates the center of gravity for the area under the curve



Fuzzy Control Systems

- The inference engine in a fuzzy system consists of linguistic rules
- The linguistic rules consist of two parts:
- an antecedent block (the conditions), which consists of the linguistic variables
- a consequent block (the output)

FUZZY ALGORITHM

- Algorithm that includes at least some fuzzy instructions, such as conditional or unconditional action statements
- Fuzzy conditional statement (A → B)
- Conditional statement in which A and/or B are fuzzy sets e.g. IF temperature is *hot* THEN fan speed is *high*
- Defined in terms of a fuzzy relation between the respective "universes of discourse" of A and B (compositional rule of inference) e.g. relation between temperature groupings and fan speeds

Types of Fuzzy Algorithms

A- Definitional algorithms

- Define a fuzzy set or calculate grades of membership of elements, e.g.:

  handwritten characters (what could an "M" look like?)

  measures of proximity (what counts as *close*?)

B- Generational algorithms

Generate a fuzzy set e.g. an arbitrary sentence in some
natural language that needs to be grammatically valid
according to various rules

C-Relational algorithms

Describe a relation between fuzzy variables

Can be used to approximately describe behaviour of a system

e.g. in our cooling fan example, describing the relation between the input variable (temperature) and output variable (fan speed)

D-Decisional algorithms

Approximately describe a strategy for performing some task, e.g. approaching a set of traffic lights (should we slow down, stop or proceed at current speed?)

navigating a robot towards a goal while avoiding obstacles

Applications of Fuzzy Logic

Control Systems

1-Consumer systems

2-automatic transmissions

3-washing machines

4-camera autofocus

Industrial systems

1-aircraft engines

2-power supply regulation

3-steam turbine start-up

E-Artificial Intelligence

1-Robot motion planning
2-Image segmentation
3-Medical diagnosis systems

Why use fuzzy logic for control?

Simple systems:

Low development costs

Low maintenance costs

Complex systems:

Reduced run-time

Reduced search space for efficient optimization

How Can Fuzzy Logic Achieve This?

Fuzzy logic:

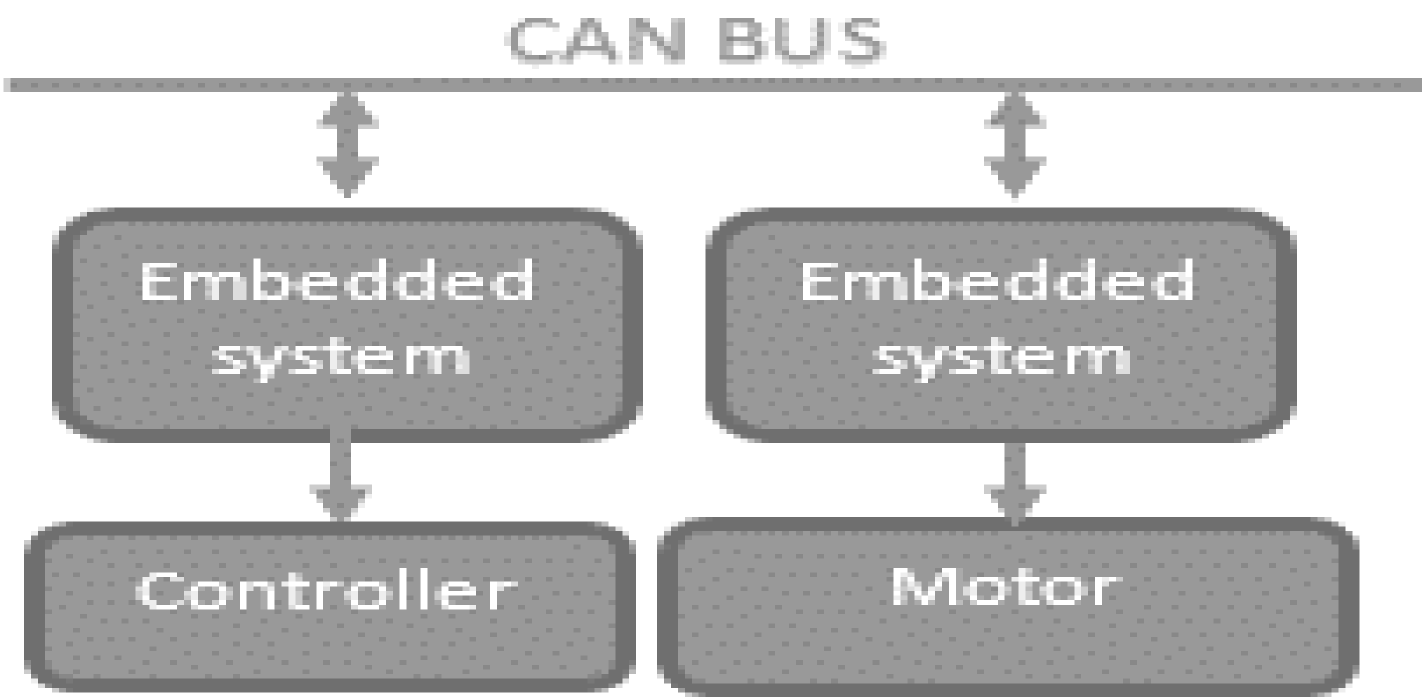
- Is used to quickly translate from expert knowledge to code
- Expert knowledge reduces the search space when optimizing the system

FUZZY CONTROL SYSTEM DEVELOPMENT

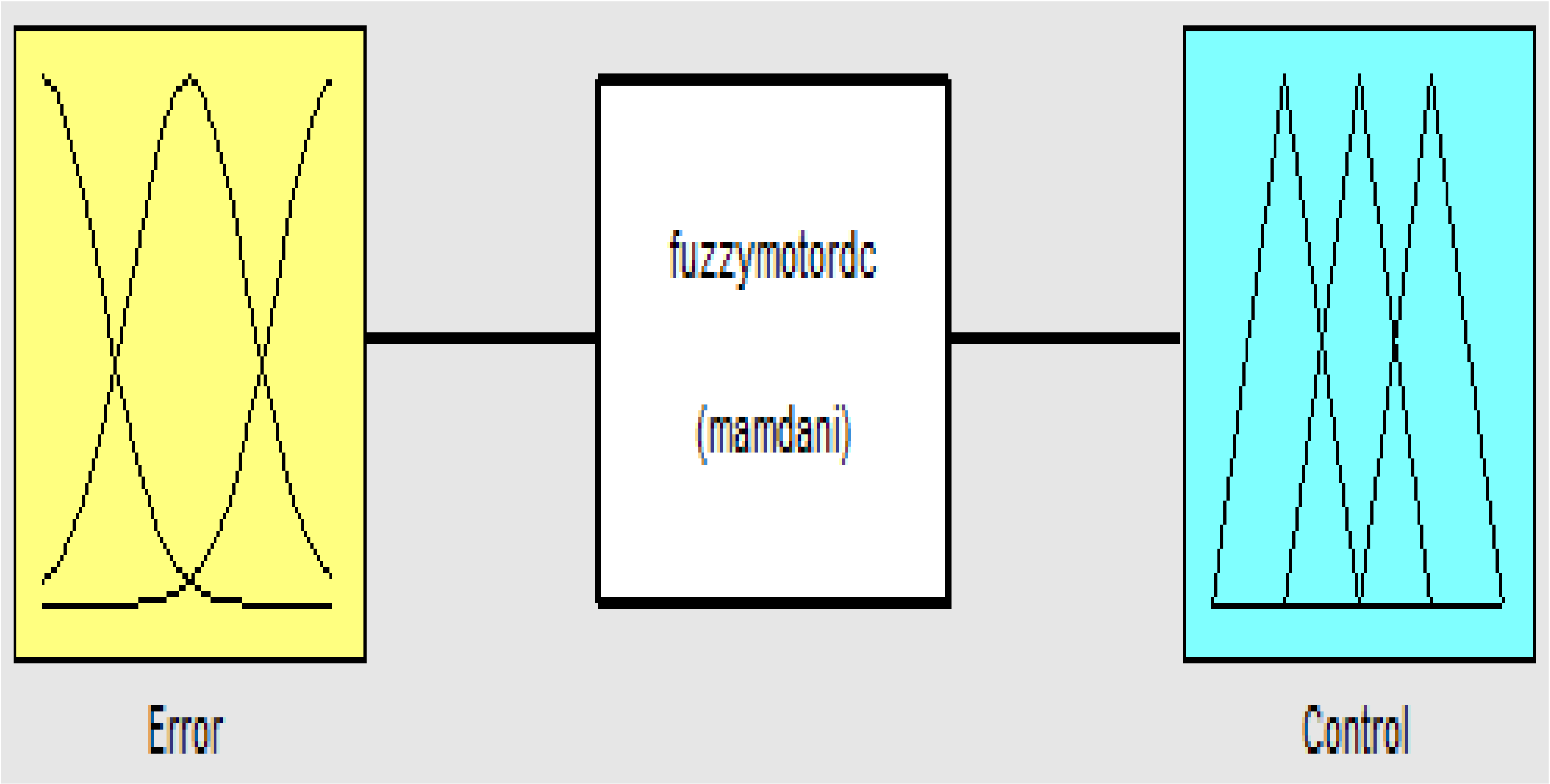
1. Identify performance measure
2. Select input/output variables
3. Determine fuzzy rules
   *Talking to an expert
   *Data mining
4. Decide on membership functions for the fuzzy variables
5. Tune membership functions and/or rules

APPLIED EXAMPLE

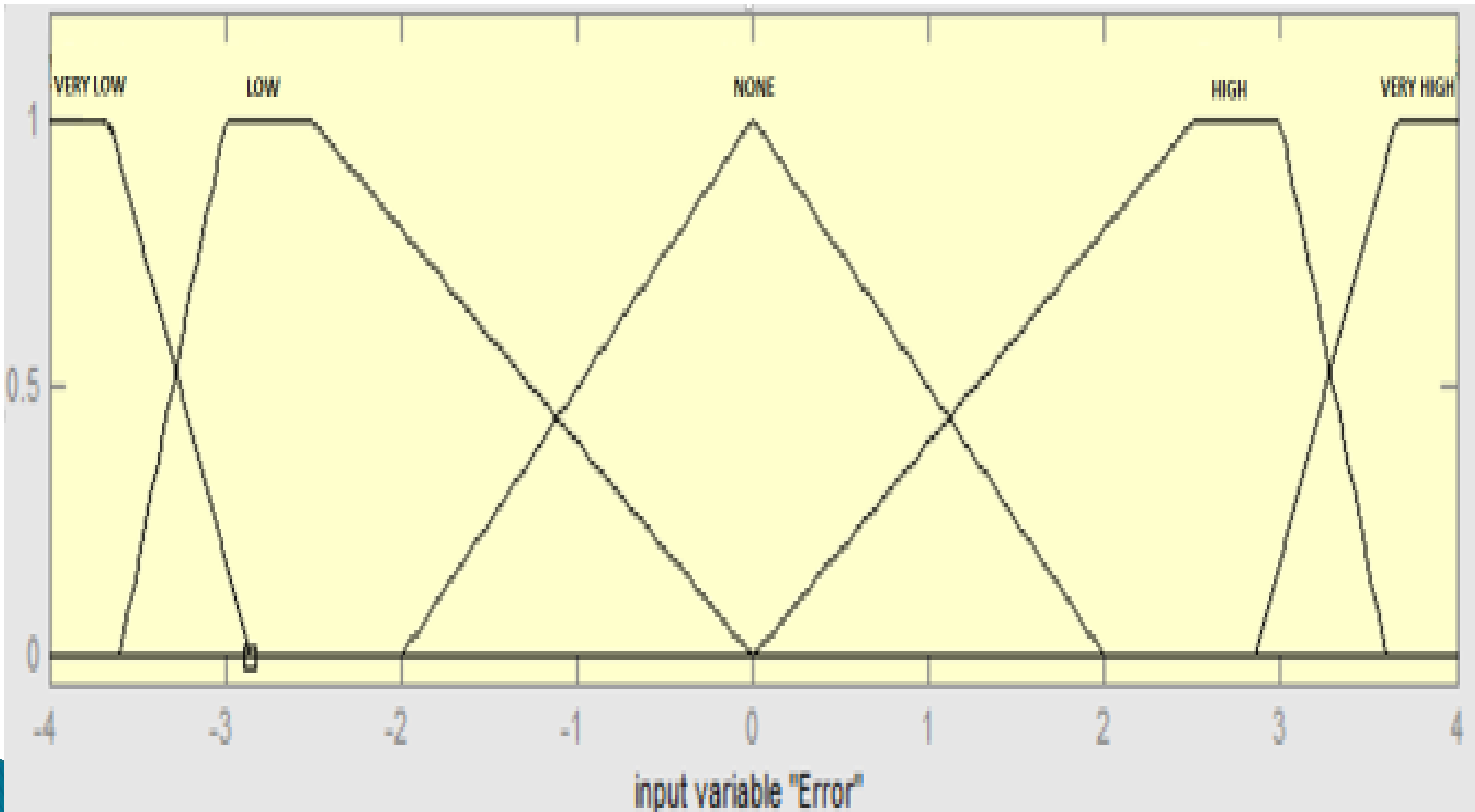
Control system using HIL, PID and Fuzzy Logic with Rapid Prototyping



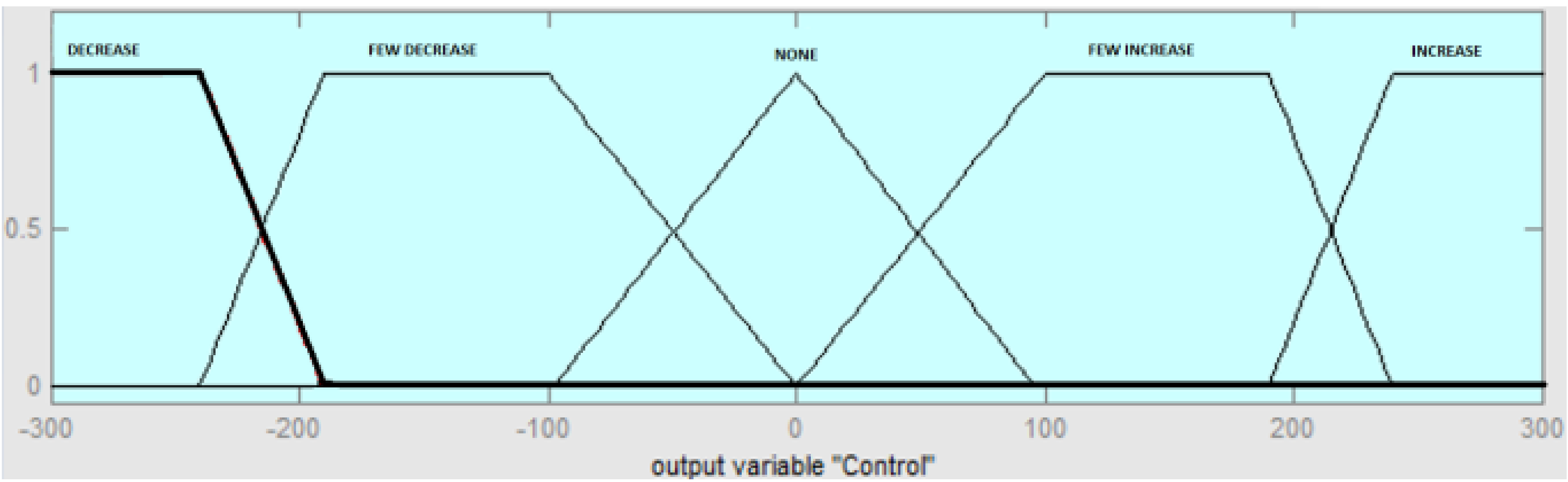
Fuzzy Control Structure



INPUT FUZZY CONTROL



OUTPUT FUZZY CONTROL



CONTROL RULES

Rules

- 1. If (Error is very low) then (Control is Decrease)
- 2. If (Error is none) then (Control is none)
- 3. If (Error is very high) then (Control is Increase)
- 4. If (Error is low) then (Control is Few Decrease)
- 5. If (Error is high) then (Control is Few Increase)

INFERENCE PROCESS TO ZERO INPUT ERROR